Vagueness and Ambiguity

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Arne Naess and Ingemund Gullvåg

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1. Introduction

The terms vagueness, indefiniteness, unspecificity, ambiguity, and related terms like equivocation, hominy, and polysemy have been used in various senses to distinguish kinds of indeterminacy in ordinary discourse. Such notions of indeterminacy are sometimes constructed as pragmatic concepts, sometimes as semantic ones. These terms are themselves used in different ways. In this articles we distinguish pragmatic/semantic roughly as Charles Morris and Rudolf Carnap did: pragmatic refers to concepts and studies abstracting from actual use, taking language as a system of relations between expressions and meanings. Commonly, an expression is said to be ambiguous in so far as it can be interpreted in several different ways, can be taken in several different senses; but it is said to be vague, given a definite sense of the term. An expression is said to be vague to the extent that there may arise doubts about its applicability in particular cases. There are recognized borderline cases where there is no right answer to the question whether the expression or negation is applicable. For example, bald is a vague word; we cannot draw one definite sharp borderline between baldness and non-baldness. There will be a large fringe of borderline cases where we cannot say definitely whether the person is “bald” or “non-bald.” The word bald is not, in this case, ambiguous. It has a certain sense; but used in this sense we cannot draw a precise limit to its field of true or correct application—its extension. Vagueness, then, is recognized diffused delimitation of the extension of an expression; an expression is vague, for a given sense, in so far as the
limits of its extension are far from sharp, by virtue of the existence or possible existence of a large area of borderline cases.

An ambiguous expression can be vague for one of its senses but not for another one. The word *function* may be an example: used in the mathematical sense it is presumably not vague, whereas in non-mathematical uses it may be comparatively vague. Ambiguity as well as vagueness are distinct from the lack of specificity. The statement (T), “There is a man at the door,” does not specify characteristics of the man, whether he is tall or short, young or old, shabby or well dressed, what his nationality is, or his name, etc. The statement fails to specify such information, and may be said to be correspondingly unspecific. But this is not to say this it is ambiguous. Nor is its lack of specificity the same as vagueness. The statement (U), “There is a tall bald man at the door,” is more specific than (T), but scarcely less vague. On the contrary, it might be said to be more vague, since it has at least a borderline case which is not a borderline case for (T); whereas any borderline cases for (T) would be one for (U). One borderline case for (U) but not for (Y) would be a situation where we would be uncertain whether to describe a man at the door as bald or non-bald. Discourse is sometimes described as “unclear,” “indefinite,” “vague,” etc., where these expressions do not refer to diffuseness of limitations of extensions of expressions, but rather to the lack of intelligibility of the discourse, or the superficiality or wanting perspicuity of the thoughts expressed in it. From empirical studies of use, intuitions about acceptable and unacceptaable expressions, and rules of use announced by users of a language, consistent systems of rules of use announced by users of a language, consistent systems of rules are formulated, for example, grammars. If a system gains authority, usage violating that system is called incorrect. But persistent ‘violation’ among broad or influential groups of people motivate changes of the system or adoption of a new one as authoritative. Then the judgments of incorrectness change accordingly. This affects what is said to be ambiguous. Systems are always fragmentary in relation to the richness of variations of usage. Therefore what is said to be ambiguous in relation to a proposed system may not be ambiguous in (actual) use, and vice versa.

## 2. Ambiguity

### 2.1. Ambiguity, pragmatics, and semantics

Ambiguity is sometimes defined as the *actual* use of an expression to express several different meanings, or else as the possibility for an
expression to be used or understood in several different ways. When meaning variance in actual use in definite context is stressed, ambiguity is constructed as a pragmatic concept rather than a semantic one. In other cases, ambiguity is constructed as a semantic notion, so that a syntactical unit (a morpheme, word, sequence, sentence, etc.), regarded as part of a linguistic system in abstraction from use, can be said to be ambiguous. The semantic property of ambiguity of an expression may be taken to explain the actual occurrence of different ways of using or understanding the expression. For example, the semantic ambiguity of the word *refuse* may be supposed to contribute to explaining why the text “Garbage service. We accept bottles and refuse paper” may be understood in two different ways. Ambiguity is sometimes defined in a way that presupposed a concept of sense (in the Fregean way) as distinct from extension.

Ambiguity consists in one expression’s having several different senses or being interpretable as expressing several different senses. Arne Naess developed a theory of ambiguity and interpretation as pragmatic concepts, a theory that does not presuppose a concept of ‘sense’ or ‘meaning’ but only concepts of ‘synonymity’ (meaning the same as) and ‘non-synonymity’, with substitutability in actual use as a criterion of synonymity. A given pair of expressions may be synonymous in some situations but not in others, and for some persons but not all, being substitutable in some cases but not all cases. In terms of the concepts of synonymity and non-synonymity, concepts of ambiguity, interpretation, and preciseness (in Naess’s sense) may be defined. Roughly, we may say that an expression (as a type) is ambiguous if and only if it has non-synonymous instances (tokens); one expression is an interpretation of another one if and only if both are (would be) synonymous for many persons in many (types of) situations; hence, a plausible interpretation of a sentence may be said to indicate a frequent use of it. Finally, one expression is a precision of (more precise than) another if and only if the set of plausible interpretations of the former is a proper subset of the plausible interpretations of the latter. Hence, a precision of a sentence may be said to eliminate some of its ambiguities in ordinary usage without adding any new ones. Thus we may note that “more precise than,” as here defined, means “less ambiguous than.” This should be distinguished from the use of “more precise than” to mean “less vague than,” a notion discussed below.

**Levels of ambiguity**

In the following, we shall distinguish between semantic and pragmatic levels of ambiguity, in the sense of the possibility of understanding an expression or utterance in several different ways, each compatible with
the rules of language. Ambiguity of an expression, on the semantic level, is construed as being rooted in the lexical or syntactical rules of the language: different, \textit{prima facie} equally applicable rules attribute different meanings to the expression.

2.2.1. Lexical ambiguity is ambiguity of single words: a word may express different meanings in different contexts, for example, in different sentences and in different situations. For example, consider the word “cape” in the sentences: “The ship passed the cape,” and “Mrs. McKinsey wore a cape.” In lexical ambiguity, the same sound (word) is associated with several different lexical or semantic, rules, so that it may mean different things in different contexts or situations.

Constraints on ambiguity may be imposed by a context. The systematic or lexical ambiguity of a word considered in isolation may be eliminated when the word is used in a sentence, as in the above examples. One meaning of the word makes the sentence intelligible as a whole whereas others do not, or can be made to do so only with an effort of the imagination. In the sentence “Mrs. McKinsey wore a cape,” the sense “promontory” of the word “cape” would hardly contribute to an intelligible sentence meaning. We may suppose that the sentence, as a context for the word, imposes constraints on the interpretation of the word so as to exclude some, perhaps all but one, of its possible senses. Some of these constraints may have to do with the intelligibility or probability of the state of affairs described by the sentence.

\textbf{Amphiboly and structural ambiguity}

Since Aristotle,\textsuperscript{5} “amphiboly” (ἀμφιβολία) has often been used to denote ambiguity of whole sentences.\textsuperscript{6} An example borrowed from Aristotle: “I wish that you the enemy may capture.” Another example: “The peasants are revolting.” Ambiguity of a sentence may be due to a residue of lexical ambiguities of one or more of the words in the sentence, as in “The bank was the scene of the crime,” or to the possibility of grouping the words together in different ways, that is, structural ambiguity. Examples: “The far major’s wife was fond of macaroni.” “I heard about him at school.” “The shooting of the hunters was terrible.” “He had two adult sons and one daughter in a nunnery.” An important form of structural ambiguity is ambiguous cross reference, when an expression, for example a pronoun, refers to something mentioned before, but the context does not make clear which of several possible referents is intended. An example: “When Napoleon ordered Marechal Ney to attack, he was very angry.” The prophecies of
ancient oracles were often structurally ambiguous, in such ways that the prophecy might be claimed to be true regardless of what turned out to happen. For example, a reply of an oracle was “Pyrrhus the Romans shall, I say, subdue.”

2.2.4. Ambiguity, surface structure, and deep structure
The new paradigm of generative grammar that replaced taxonomic structuralism in linguistics conceives of grammatical analysis as the construction and testing of theories about the speaker’s internalized linguistic competence. According to Noam Chomsky, a grammatical theory should throw light on such facts as: we can understand new sentences, we can distinguish grammatical and ungrammatical sentences even when we have never heard them before, and we have acquired a great number of intuitions about linguistic form. The fact that we can understand ambiguous sentences, recognize the ambiguities and disambiguate such sentences, is among the facts to be accounted for by linguistic theory. The need to account for syntactic ambiguity and competent speakers’ ability to disambiguate such sentences is among the facts to be accounted for by linguistic theory. The need to account for syntactic ambiguity and competent speakers’ ability to disambiguate such ambiguity was emphasized. However, on the level of syntax there seem to be two types of ambiguity, a more superficial one and an underlying one. The existence of the latter, presumably exemplified by “The shooting of the hunters was terrible.” led to the assumption of a deep structure level that grammars should account for. Chomsky’s analysis seems implicitly to take account of ambiguities at three distinct linguistic levels (lexical, surface structure, and deep structure ambiguities). Deep structure ambiguities depend to a greater extent on underlying logical relationships between items in the sentence. Such ambiguities depend to a greater extent on underlying logical relationships between items in the sentence, as is supposedly the case with “Visiting relatives can be a nuisance.” Thus, ambiguity has been an important consideration in the development of modern generative grammatical theory. Adequate specification of the readings for ambiguous sentences was taken as a condition for the construction of wellformed grammars.

2.2.5. Pragmatic ambiguity
Semantically or syntactically ambiguous sentences may often be disambiguated by taking into account greater units of verbal context or non-verbal circumstances of situations of use. We may suppose that when pragmatic circumstances, such as users and wider contexts and situations of use are considered, more severe constraints are imposed on the interpretation so as to exclude some, perhaps all but one, or maybe
even all, possible consistent or understandable meanings of the sentence or passage. But, an utterance or expression may remain ambiguous even when such pragmatic factors are taken account of. We may call this ambiguity on the pragmatic level. A string of words, e.g., a sentence or passage, used by someone in some situation, may be interpreted in several different ways, because the meaning is undetermined by linguistic (lexical and structural) rules and context or situation of use. For example, a passage in Spinoza’a Ethica may be undetermined, even if the rules of Latin and the whole verbal context and situation of use are taken into account, including general knowledge of Spinoza’s life and philosophy; several different interpretations may be equally plausible.

Different Spinoza-experts may defend different interpretations of the text, and no objective factor is found that would impose a further constraint so as to eliminate one of the contending interpretations. More trivial examples of pragmatically ambiguous expressions occur in newspaper articles, political speeches, etc. Consider, e.g., the sentence “Private initiative must be introduced into health care,” occurring in a short newspaper review of a political debate which gives no further clues as to how the sentence is to be interpreted. In the case of pragmatic ambiguity, we may distinguish between ambiguity with respect to the illocutionary force of an utterance, and ambiguity with regard to its context or meaning. But in the case of the latter, we may also distinguish between ambiguity with respect to what the speaker wishes to express explicitly and ambiguity with respect to what transpires from the utterance by virtue of innuendo, pragmatic implicature, etc. A leading idea of the preceding paragraphs is that the range of possible meanings of expressions or strings of expressions is narrowed down by constraints imposed by linguistic (lexical and structural) rules, pragmatic conventions, e.g., dialogue conventions, verbal contexts and situations of use, and perhaps considering of intelligibility or probability of (described) situations. Such constraints may narrow the field of possible meanings down to one, for a wide set of interpreters. Ambiguity on any level, lexical, structural or pragmatic, may be construed as under determination of meaning by constraints imposed at (up to and including) that level: two or more different meaning-options are left open.

The ubiquity of ambiguity

How pervasive is ambiguity? The question itself is highly ambiguous. One answer would be: It depends on the list of possible meanings to which we refer. If the list is short and crude we may find a sentence
unambiguous, for instance, the sentence “The Earth is surrounded by a
field of gravitation.” But if we go into precisions of the word
“gravitation” the sentence may be said to permit different
interpretations. When we construe an expression as a part of a language
in the sense of a linguistic system, we may be said to permit different
interpretations. When we construe an expression as a part of a
language in the sense of a linguistic system, we may be inclined to
conceive of the expression as having one and only one definite sense,
unless it is associated with several different meaning-rules in the
system, like “bank” or “band” or “cape,” or alternative structural rules.
In the case of such expressions, we may be inclined to say that the
system in question is such that the words or sentences are ambiguous.

But ‘language’ may be construed as a kind of idealized abstraction from
actual instances of use involving many kinds of variations of use, as
done in the quotation: “A grammar of language purports to be a
description of the ideal speaker-hearer’s intrinsic competence.” When
we think about ‘Language’ or ‘Grammar’ we may disregard many kinds
of variations of use. Such factors as we then overlook, may be the ones
we have to take into account when we try to understand or interpret
particular utterances or texts. In the linguistic community what is talked
of as correct shows socially and politically interesting relations to class
and social hierarchical relationships in general. The idea of correctness
as a tool of domination has been extensively studied.

But in daily life we are normally more interested in what someone may
actually have sought to express, or how some group of persons actually
understand an utterance or text, than in finding out what is ‘correct or
incorrect use of language’, or what characterizes ‘the ideal speaker-
hearer’s intrinsic competence’. Normally, we try to clarify disturbing
ambiguities and vagueness, and we succeed. The utility of expressions
is due to their having ‘dictionary’ as well as ‘contextual’ sense (that can
vary in different situations); words must be recognizable to any person
with linguistic competence, but each user seeks to express the particular
nuance of meaning that fits into the context where he uses the word.
That linguistic expressions can be used in many varying ways,
according to situations and needs, has been called the “efficiency of
language.” Ambiguity, according to Jon Barwise and John Perry, is
simply a species of efficiency of language. But when language is used
in particular cases, there may arise insecurity and misunderstandings
with respect to contextual meanings; and they cannot be eliminated
simply by appealing to ‘language’ or ‘correct usage’. We may say, in
short, that utterances and texts are often ambiguous in the sense that
pragmatic ambiguity is present. However, ambiguous sentences are not

62 The Trumpeter
necessarily perceived to be so. When ambiguities are obvious, this is often a source of humour as in puns. Unnoticed ambiguities, on the other hand, may be sources of fallacies in reasoning, and of spurious agreement or disagreement in discussion.

**Fallacies due to ambiguity**

Aristotle in *De sophisticis elenchis* distinguished between fallacies dependent on language and fallacies ‘outside language’. The former kind are fallacies arising from ambiguity. The simplest of these are fallacies of equivocation, fallacies due to lexical ambiguity. Fallacies of amphiboly are fallacies arising from double-meaning of sentences. One version of the so-called ‘four term fallacy’ is a fallacy of equivocation of the middle term in a syllogism, violating the rule that every syllogism has three, and only three, terms. An example given by the late Sydney Herbert Mellone: “All metals are elements. Brass is a metal. Therefore, brass is an element.” Mellone comments that here, the middle term, “metal” is used in two different senses, in one case about pure metals (which are elements), in the other, about pure metals or alloys. Clearly, this is a case of fallacy of equivocation; but such fallacies of equivocation of the middle term are often classified as cases of the four term fallacy. A possible justification for this might be the claim that if a term is used ambiguously, it is really two terms. But such a theory of ambiguity seems dubious since it would appear to have the consequence that there are really no ambiguous expressions, but more or less unsurveryable multitudes of like-sounding and similar-looking unambiguous expressions. A more acceptable justification might perhaps be that in a syllogism with ambiguous middle term, actually four rather than three *concepts* are involved. Charles Leonard Hamblin, however, criticizes the common classification of fallacy equivocation of the middle term as the (formal) four term fallacy.

The middle term cannot be equivocal unless it is *one* term with *two* meanings. If there are really four terms we have a formal fallacy, independently of whether any term is equivocal; if we have an essentially equivocal term there is a fallacy of the Aristotelian variety /depending on language/ whatever the formal shape of the argument. 16

Fallacies may occur in ethical argumentation because key words used in such arguments may be ambiguous as between normative and descriptive senses. A famous example is found in John Stuart Mill’s argument for the conclusion that “happiness is desirable, and the only
thing desirable, as an end,” where he shifts between a descriptive and a normative sense of “desirable.”

**Pseudo-agreement and –disagreement**

Ambiguities may give rise to pseudo-agreement or—disagreement, verbal agreement or disagreement that does not correspond to agreement or disagreement in fact. This kind of misunderstanding may be quite common: in any case a large part of daily personal discourse as well as political discussion is conducted without sufficient guarantee that the participants really are in accord or discord on what they believe they agree or disagree about. An example of a dialogue that shows symptoms of pseudo-disagreement at a certain stage: (1) *A*: “A welfare society cannot exist without socialism.” (2) *B*: “I disagree. A number of European countries have been welfare societies even under non-socialist governments.” (3) *A*: “I did not mean that. I meant that a welfare society cannot be developed in a country unless a social ideology has a strong influence in that country.” (4) *B*: “Oh, I see. Well, if that was what you meant, I agree.” At stage (2) in the dialogue, *B* declares his disagreement with *A*’s remark (1), i.e., there is ‘verbal disagreement’. But at stage (3), *A* rejects the interpretation of (1) on which *B* based his disagreement; and restates his opinion, presumably in a more unambiguous way, i.e., ‘more precise’, in Naess’ sense.19 In (4), *B* expresses his agreement with what he takes *A* to mean in (3). Levels (3) and (4), then, suggest that the disagreement in (2) was merely verbal and not real; hence a pseudo-disagreement. However, it is to be noted that if the dialogue continues, it may very well at later stages show symptoms that the agreement at level (4) was merely verbal and not real, hence a pseudo-agreement. Hence, any such judgment about real/pseudo agreement/disagreement must be provisional and tentative.20 Dagfinn Føllesdal21 suggests that Willard Van Orman Quine’s rejection of the distinctions analytic/synthetic and meaning/belief may inspire doubts about the tenability of the distinction between verbal and real agreement/disagreement.22

**Ambiguity as fault or virtue**

Vagueness, unspecificity, and indefiniteness are sometimes regarded as relatively harmless or, in the case of vagueness, unavoidable aspects of ordinary language and discourse. Ambiguity, on the other hand, is often described as undesirable and harmful but remediable; this is a tradition that goes back to antiquity. Marcus Tullius Cicero was concerned with the fact that misunderstandings due to ambiguity may be utilized in
dishonest ploys of persuasion. Use of ambiguous words should be avoided, or one ought to display their different meanings. A number of other philosophers and logicians in antiquity and medieval times regard ambiguity as harmful. After the Renaissance, this tradition has continued. For example, Pertus Ramus wrote that “the fault of ambiguity . . . is a common fault of all speech.” Blaise Pascal criticized the dishonest use of ambiguous expressions where one pretends to use them in a sense that is different from the meaning that one actually intends. Similar views have been expressed by more recent semanticists and philosophers of language. Some, for example, Herman Tenessen, have pointed out that in debates one may sometimes hear a participant alternating between different interpretations of one of his statements, on the one hand interpretations which make it safe (but quite trivial and uninteresting), on the other hand interpretations which make the statement interesting and consequential (but uncertain or doubtful). In order to emphasize the importance of the statement, or draw consequences from it, the speaker takes recourse to the more daring interpretations. In this way, the speaker can create the impression that his statement is interesting as well as tenable.

An example from a discussion between politicians about possible savings in the public health services: The politician A states: “It should be permitted to reduce the offer of therapy to patients with chronic mental illness, since (t) this group cannot be cured anyway.” The expression “patients with chronic mental illness” is ambiguous; it may mean either ‘patients with incurable mental illness’ or ‘patients of mental illness with long duration’. The former interpretation makes (T) synonymous with (U) “the group of patients with incurable mental illness cannot be cured anyway,” and hence trivial (or analytic, if one accepts the notion of analyticity). The other interpretation makes (T) synonymous with (V) “the group of patients with mental illness of long duration cannot be cured anyway.” (V) is not trivial or analytic, but will be quite controversial and is quite likely false. In his speech, A draws a conclusion from (T) (“It should be permitted to reduce the offer of therapy to patients with chronic mental illness”) and draws the further conclusion that considerable amounts can be saved on the health-budget by reducing the offer of therapy to the whole group of patients with mental illness of long duration. Here, he applies the non-trivial interpretation of (T). But when A is attacked by the politician B who says that A’s view is unreasonable and unacceptable in a modern welfare society, A defends himself by claiming that by “patients with chronic mental illness” he merely meant ‘patients with incurable mental illness’, and the offer of therapy must obviously be wasted on incurable patients. In spite of the possibility of abuse and harmfulness of
ambiguity, some contemporary authors, however, emphasize that ambiguity is simply an aspect of the general ‘efficiency’ of language, the fact that linguistic expressions can serve many different purposes in different contexts and situations, and that is precisely this ‘efficiency’ that makes language so immensely useful.\textsuperscript{25} Ambiguity may be harmful, but it is not necessarily so under all circumstances. In poetry, multiple meanings of words may be an asset, they make for greater richness of associations.\textsuperscript{26}

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**Intolerance of ambiguity**

It has been suggested that linguistic ambiguity is related to ambiguity in a very wide sense, for instance emotional ambivalence, and reversibility in visual perception of figures like reversibility in visual perception of figures like the duck-rabbit, the Necker cube, the Peter-Paul goblet, etc.

It has been claimed that general mental rigidity (low ability to restructure the cognitive-perceptual field) is related to low ability to deal with emotional ambivalence as well as low ability to recognize and discriminate between different meanings of ambiguous linguistic expressions. Psycholinguistic research has shown individual differences in the ability to detect and resolve ambiguity.\textsuperscript{27} The notion of a connection between mental rigidity, low tolerance of ambivalence, and low ability to handle ambiguity led to the assumption, in social psychology, of general ‘intolerance of ambiguity’ as a character trait of ‘authoritarian’ persons.\textsuperscript{28}

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**Vagueness**

Vagueness is commonly defined in terms of the existence or possible existence of borderline cases, and a borderline case for an expression $P$ is an object or state of affairs $x$ such that the affirmation as well as the denial of $P$ in relation to $x$ is essentially doubtful. The notion of essential doubtfulness is distinguished from uncertainty due to lack of knowledge of facts. A distinction is sometimes drawn between extensional and intentional vagueness. An expression is extensionally vague if it has some actually existing borderline case; but it is intentionally vague if it is logically possible that it could have some borderline case.

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**Vagueness as a pragmatic notion**
Charles Sanders Peirce defined “vague” as “indeterminate in intention,” and thus seems to differ from those who construe vagueness as indeterminateness of extension. On one interpretation, Peirce characterizes what has sometimes been called indefiniteness of intention or low depth of intended meaning. Peirce says that a proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. By intrinsically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker’s habits of language were indeterminate; so that one day he would regard the proposition as excluding, another as admitting, those states of things. Yet this must be understood to have reference to what might be deduced from a perfect knowledge of his state of mind; for it is precisely because these questions never did, or did not frequently, present themselves that his habits remained indeterminate.

Vagueness in this sense would be a pragmatic concept. Carl Gustav Hempel also construed vagueness as a pragmatic concept.

. . . the vagueness of a symbol consists in the existence of a speaking habit among the users which involves the occurrence of . . . variations in the application of the symbol . . . Vagueness is strictly semiotic: there is no analogue to it on the purely syntactico-semantical level . . .

Hempel’s motivation for this view has to do with problems associated with applying logic to vague expressions—problems suggested, for example, by Bertrand Russell.

Russell on Vagueness

Russell, however, defined vagueness as a semantic concept. Russell opposed “vagueness” to “precision.” According to him, there are “characteristics which can belong only to a representation, of which language is an example. They have to do with the relation between a representation and that which it represents.”

According to Russell,

a representation is vague when the relation of the representing system to the represented system is not one-one, but one-many. For example, a photograph which is so smudged that it might equally represent Brown or Jones or Robinson is vague.

As Bertil Rolf remarks, one problem with Russell’s definition is that it seems to equate vagueness with generality. Further, as Rolf points out,
it is doubtful, whether Russell’s semantic framework contains the means necessary for effecting a distinction between vagueness and ambiguity. A third problem for Russell’s definition is that vagueness as ordinarily conceived is intimately connected with the existence of borderline cases as a criterion of vagueness, but his definition makes no mention of this.

### Vagueness and logic

If, as Russell claims, all language is vague, and if standard logic applies only to nonvague expressions, it is inappropriate to actual discourse and reasoning. Nevertheless standard logic seems applicable in many cases where we use vague expressions. But the problem of reconciling the requirements of standard logic with the vagueness of ordinary expressions or concepts has engaged many logicians and philosophers, and many have either denied or seriously doubted that the two are reconcilable, e.g., Gottlob Frege, Peirce, Russell, Alfred Jules Ayer, Peter T. Geach. Peirce wrote that the vague “might be defined as that to which the principle of contradiction does not apply.” Others have doubted or denied the applicability of the principle of excluded middle to vague concepts.

**Tertium non datur?**

Frege pointed out that the law of excluded middle is merely another version of the requirement that the concept (in extension) be sharply delimited, i.e., not vague. According to Frege, a concept-word stands for a concept if and only if it clearly determined an extension: for any object, a corresponding truth value for the word is determined with no uncertainty, hence there are no borderline cases. Clearly, Frege wanted to restrict the use of the expression “concept-word” to nonvague terms since vague terms would seem to violate the requirements of logic. It is therefore not surprising that Russell held that vagueness causes problems for the law of excluded middle.

“The law of excluded middle is true when precise symbols are employed, but it is not true when symbols are vague, as, in fact, all symbols are.”

Since traditional logic presupposes the truth of the law of excluded middle, traditional does not apply to our language. Max Black and Hempel, in their papers on vagueness, tried in various ways to overcome what Black called Russell’s “sacrifice of logic.” Black tried to establish a logic taking account of the vagueness of symbols, with classic logic as a limiting case for nonvague symbols. Hempel tried to
drive a wedge between vagueness and logic by constructing vagueness as a pragmatic notion that cannot be incorporated into a syntactic-semantic system, where logic belongs. William P. Alston suggests that the problem of vagueness can be handled by rejecting the law, by denying that in a case where truth or falsity cannot be pronounced we have a statement, or by taking the law to apply only to an ‘ideal’ language.

3.3.2 σωρίης and φαλακρός
A number of paradoxes have been known by these names since antiquity. Diogenes Laertius ascribes them to Eubuleides of the Megarian school, a contemporary of Aristotle. The σωρίης, or ‘heap’, is the argument that there can be no such thing as a heap of sand since one grain is not a heap and adding one grain to something that is not a heap is never sufficient to turn it into a heap. The φαλακρός or ‘bald head’ is a similar demonstration that no one can have a head full of hair. Such paradoxes can be generated by any process of gradual change associated with dichotomy (e.g., “heap/non-heap”). The problem with such a paradox is that we seem to have good reasons for accepting the premises as true and rejecting the conclusion as false; nevertheless, the argument seems valid. In recent discussions, various solutions to this quandary have been proposed in fuzzy logic, theories of supervaluations and proposals for metalanguages which take all positive ascriptions of vague predicates to be false.

Supervaluation
People’s actual beliefs are in many cases neither simply true nor simply false. In such cases, we have two possibilities. We may either regard them as entirely without a truth value, or we may ascribe unusual truth values to them when they do not have the usual truth or falsehood as values. In theories of supervaluation, the former alternative is developed; in fuzzy logic, the latter. Theories of supervaluation have been proposed, e.g., by Kit Fine, Hans Kamp, and Marian Przełęcki. Roughly, the idea in the supervaluation approach is that a vague belief’s or proposition’s truth is its supervaluation, which is a function of the proposition’s tentative ordinary (classical) truth valuations. A tentative valuation is the ordinary truth value the proposition would have if it were made precise (= nonvague) in some particular way, so as to rule out all borderline cases. For each way of making the proposition precise, we get a new tentative classical valuation for that proposition, indicating whether the proposition, as thus made precise, is true or false. If every way of making the proposition precise makes it (classically) true, all of its tentative valuations will be true. If every precise version of the proposition is
false, all of the tentative classical valuations are false. Otherwise, we get a mixture of tentative valuations. The supervaluation of the vague proposition is truth if and only if all the tentative valuations are true; falsity if and only if they are all false; and undefined otherwise. On the theory of supervaluations, one of the premises of the version of Sorites mentioned above is false: “Adding one grain to something that is not a heap is never sufficient to turn it into a heap.” This is false, since the following would be true for each way of making it precise: “There are cases such that adding one grain of sand to something that was not a heap of sand before, would be sufficient to turn it into a heap.” One presumed advantage of the supervaluation approach is that the theorems of classical logic remain logically true on this interpretation, since the theorems will always be true on every tentative classical valuation. Kenton F. Machina questions this supposed advantage. He claims that although the supervaluation approach preserves all the classical tautologies, it does not preserve all the classical rules of inference. He suggests that the supervaluation approach runs afoul of what seem to be reasonable requirements on what can count as valid inferences.

**Fuzzy Logic**

The basic ideas of fuzzy logic apparently stem from Black’s paper on vagueness. In fuzzy logic, in cases where vague propositions are neither true nor false, they are assumed to take on ‘unusual truth values’ thought of as degrees of truth and falsehood. This approach has been advocated, e.g., by Lotfi Asker Zadeh, Joseph Amadee Goguen and Machina. The assumption is that there are infinitely many truth values which are generally represented by the unit interval from 0 to 1; where 0 represents complete falsity and 1 complete truth. Machina claims that Jan Łukasiewicz’s system, $L_\alpha$ is well suited to serve as a logic of vagueness. A qualification theory and a set-theoretic semantics is provided by means of a generalized set theory described by Zadeh and developed by Goguen. This set theory differs from ordinary set theory by allowing the set membership to admit of degrees, providing so-called “fuzzy sets.” To each predicate letter in the calculus, a fuzzy set is assigned as its extension, so that some members of the domain of interpretation are ‘in’ the extension of a given predicate letter only to a limited extent. Fuzzy logic claims to solve the sorites paradox by denying that the sorites argument is completely valid. Each step of the argument is slightly invalid, so the truth guaranteed slowly leaks away as we try to carry it along the chain. But fuzzy logic also claims to explain the deceptiveness of the sorites argument. Rolf claims that the theory of truth presented by fuzzy logic is seriously deficient.
Variability of application and borderline cases

Max Black took over from Peirce the idea that ‘borderline case’ for an expression is definable in terms of variation concerning the application (extension) of the expression. This idea recurs in many definitions of vagueness. Rol[59] argues against this that individual variations in applications of an expression are neither sufficient nor necessary for establishing borderline cases and vagueness of this expression. A borderline case for an expression T is not one where approximately half of the speakers of the language in question affirm the expression and the rest deny it; rather a borderline case for T is one where neither affirming nor denying T can be said to be ‘correct’ and where affirmation as well as denial of T would convey information that was misleading because too definite. The application of T as well as -T is ‘essentially doubtful’ or arbitrary.

3.5 Vagueness and Ignorance

As suggested (in 3.), this ‘essential doubtfulness’ of the applicability of a vague expression is supposedly a distinct form of uncertainty due to lack of knowledge of facts. On the traditional concept, vagueness is taken as a semantic property of expressions that is independent of the speaker’s ignorance of facts. The indeterminacy is due to an aspect of the meaning of the term rather than to the current state of our knowledge. Quine’s skepticism about a distinction between matters of meaning and matters of fact might inspire doubts about the traditional concept of vagueness. Such doubts are voiced by Israeal Scheffler.[62]

Intolerance of Vagueness

It might be surmised that vagueness, like ambiguity, may be connected with broader psychological and emotional phenomena. Actually ‘black-white thinking’ suggested as a trait of the ‘authoritarian personality’, might be regarded as a manifestation of a generalized intolerance of vagueness, the need for a yes-or-no answer on any issue, the dread of intermediate ‘grey zones’ or uncertainty or undecidability, and the hostility towards any inclination to withhold judgment. This ‘generalized intolerance of vagueness’ may be related to the incapability of admitting intermediate cases such as those expressible in modal systems; e.g., the case where something is ‘adiaforon’ in the sense of neither obligatory nor prohibited, or where something is neither affirmed nor denied, or neither known to be true nor known to be false.
Selected references


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Naess, Arne. 1966, Communication and Argument. Contains elementary discussions of ambiguity and interpretation, and an outline of the theory of pseudo-(dis)agreement. (Now SWAN 7)


Rolf, Bertril. 1981. Topics on Vagueness. A collection of articles with discussions of the main philosophical points of view on vagueness. The classical articles by Russel, Black and Hempel are reviewed and criticized. Sorites paradoxes are discussed, and also the solutions proposed in fuzzy logic and theories of supervaluations. A bibliography on vagueness is appended.

Salmon, Wesley. 1948. Introduction to Logic and Critical Thinking.

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1 s. art. 92
2 s. art. 81
3 Naess, 1953, chap. I
4 s. art. 86
s. art. 15
6 cf. De soph. El., 165b 23-27; 166a 6-14; 22
7 Chomsky 1965, 4; Bever/Katz 1976, 11
8 cf. Kess/Hoppe 1981, 13 ff
9 s. art. 95
10 s. art 97
11 Chomsky, 1965, 4 s. art. 67
12 s. art. 49
13 Barwise/Perry 1983, 5; 32-39
14 Jon Barwise and John Perry, 1983, 5, 40
15 Sydney Herbert Mellone, 1902, 166
16 Hamblin, 1970, 45
17 s. art. 30
18 Mill 1962, 288
19 s. art. 47
21 Føllesdal/Walløe/Elster 1984, 200
22 s. art. 86
23 s. art. 4
24 Ramus 1964 [1543], Aristotelicae animadversions, 70
25 Barwise/Perry 1983, 40
26 cf. Empson 1965, 3
27 Kess/Hope 1981, 39
28 cf. Rokeach 1948; Frenkel-Brunswik 1949
29 s. art. 32
30 Peirce 1902, 748
31 cf. Naess 1953, 79f
32 Naess 1966, 34f
33 Peirce 1902, 748
34 1939, 170
35 1939, 190
36 Russell 1923, 85
37 Russell, 1923, 89f.
38 1981, 3
39 1981, 4
40 Russell 1923, 85
41 cf. Rolf 1981, 4f
42 s. art. 34
43 Alfred Jules Ayer, 1968, 115
44 Peter T. Geach, 1972, 85
45 CP 5.505
46 1903, 69
47 Russell 1923, 85f.
49 cf. Hempel 1939, 180
50 1967, 221
51 1925, II, no. 108
53 Kit Fine, 1975
54 Hans Kamp, 1975
Marian Przełęcki, 1969; 1976; 1977
Kenton F. Machina, 1975
Max Black 1937
Bertril Rolf, 1981; 1984
Rolf, 1981
Rolf, 1981, 35f
cf. Alston 1967, 218
Israel Scheffler, 1979, 75; cf. objections in Rolf 1981, 79
Adorno/Frenkel-Brunswik/Levinson/Sanford 1950